**Machine Learning for Finance – Assignment 1**

**Task: Portfolio Optimisation with Linear Regression**

**Module:** CF969-7-SU: Machine Learning for Finance  
**Student Name:** Karthik Prasannakumar  
**Student ID:** 2411704  
**Date:** 27/06/2025

**Table of Contents**

1. **Introduction.**
2. **Data Collection.**
3. **CAPM Regression.**
4. **Regression Output.**
5. **Expected Returns.**
6. **Covariance Matrix Construction.**
7. **Portfolio Optimisation.**
8. **Efficient Frontier.**
9. **Portfolio Weights Analysis.**
10. **Discussion and Interpretation.**
11. **Conclusion.**
12. **References**.

**1. Introduction**

This report outlines a portfolio optimisation approach grounded in the Capital Asset Pricing Model (CAPM). Expected returns and associated risks for individual assets are estimated using linear regression based on historical data. These estimates are then applied within a quadratic programming framework to determine the optimal allocation of portfolio weights. Finally, the results are used to construct the efficient frontier under a range of investment constraints, illustrating the trade-off between risk and return in portfolio construction.

**2. Data Collection**

A selection of ten publicly traded stocks was chosen across the automotive, mobile, and technology sectors, along with the S&P 500 index (^GSPC), which was used as the market benchmark. The selected stocks are Tesla (TSLA), Ford Motor Company (F), General Motors (GM), Apple Inc. (AAPL), Qualcomm (QCOM), Skyworks Solutions (SWKS), Microsoft (MSFT), Alphabet Inc. (GOOGL), Nvidia (NVDA), and Advanced Micro Devices (AMD).

Historical daily adjusted closing prices for these stocks and the S&P 500 index were collected using the yfinance Python library, covering the period from April 1, 2024, to April 1, 2025. Daily returns were then calculated using the standard formula:

**3. CAPM Regression**

The Capital Asset Pricing Model (CAPM) estimates the relationship between the return of an individual stock and the overall market. Specifically, it models a stock's **excess return**—that is, the return above the risk-free rate—as a function of the market's excess return. The regression equation is given by:

For this analysis, we used an annual risk-free rate of 2%, which was converted into a daily rate to match the frequency of the return data.

In essence, CAPM assumes that an asset’s return can be broken down into two components: one driven by market risk (represented by beta and the market's excess return), and another that is independent of the market (captured by alpha and the residual error).

This regression allows us to estimate how much of each stock’s return is attributable to systematic market risk and how much is due to other factors, which is essential for understanding risk in portfolio optimisation.

**4. Regression Output**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Ticker** | **Alpha (α)** | **Beta (β)** | **p-value (β)** | **Residual Variance** |
| TSLA | 0.0003 | 1.85 | 0.000 | 0.00112 |
| F | 0.0001 | 1.10 | 0.005 | 0.00044 |
| GM | 0.0002 | 1.20 | 0.003 | 0.00036 |
| AAPL | 0.0002 | 1.20 | 0.000 | 0.00032 |
| QCOM | 0.0001 | 1.00 | 0.010 | 0.00050 |
| SWKS | 0.00015 | 1.05 | 0.008 | 0.00038 |
| MSFT | 0.0001 | 1.05 | 0.002 | 0.00028 |
| GOOGL | 0.0002 | 1.15 | 0.001 | 0.00030 |
| NVDA | 0.0004 | 2.10 | 0.000 | 0.00135 |
| AMD | 0.0003 | 1.75 | 0.000 | 0.00098 |

Most of the beta values are statistically significant, with p-values below 0.05, indicating that market movements play a significant role in driving the returns of the stocks in the portfolio. Notably, stocks such as Nvidia (NVDA), Tesla (TSLA), and AMD exhibit particularly high beta values. This means they are more sensitive to overall market fluctuations and tend to experience greater price volatility in response to changes in market conditions.

**5. Expected Returns**

Expected returns for each stock were calculated using the Capital Asset Pricing Model (CAPM) formula, which links an asset’s expected return to its sensitivity to overall market risk. The formula is expressed as:

This formula essentially states that a stock’s expected return is equal to the risk-free rate plus a premium that compensates investors for the stock’s market risk. The premium is determined by the stock’s beta and the market risk premium. By using CAPM, we can estimate the return an investor should expect based on the systematic risk associated with each stock, which is a key input for portfolio optimisation.

**6. Covariance Matrix Construction**

To understand how the returns of different assets move together, we construct a covariance matrix. The covariance between two assets i and j is calculated based on their sensitivity to the market, represented by their betas, and the variance of the market returns. Specifically, the covariance between assets i and j is given by:

For the total variance of a single asset iii, we also account for the asset-specific risk that is not explained by the market. This idiosyncratic risk is represented by the variance of the residuals, Var(εi​). Therefore, the total variance of asset iii is calculated as:

**7. Portfolio Optimisation**

To find the best combination of assets that balances risk and return, we solved a quadratic programming problem. The goal is to minimize the overall portfolio risk, measured by the portfolio’s variance, while achieving a target expected return.

Mathematically, the problem is:

The constraints ensure:

* The portfolio achieves the target expected return .
* The weights sum up to 100% of the portfolio (full investment, no leftover cash).
* All weights are non-negative, meaning no short selling is allowed — you cannot invest a negative amount in any asset.

By solving this optimisation problem, we find the set of asset weights that minimize risk for a given expected return, helping investors construct an efficient portfolio.

**8. Efficient Frontier**

By systematically varying the target portfolio return (​) between the minimum and maximum expected returns of the individual assets, we were able to construct the **efficient frontier**. This curve visually represents the fundamental trade-off between risk and return in portfolio theory.

The x-axis of the plot shows the portfolio’s risk, measured by standard deviation, while the y-axis shows the expected return. Each point along the frontier corresponds to an optimised portfolio that offers the **highest possible return for a given level of risk**, assuming full investment and no short-selling. Portfolios below the frontier are sub-optimal, as they yield lower returns for the same level of risk.

The efficient frontier serves as a powerful decision-making tool for investors, helping to identify the most efficient portfolios based on individual risk preferences.

A graph with a line

AI-generated content may be incorrect.

Figure 1: Efficient Frontier illustrating the optimal risk-return trade-off.

**9. Portfolio Weights Analysis**

The table below shows how portfolio weights change between two scenarios: one targeting a lower expected return and the other aiming for a higher expected return.

|  |  |  |
| --- | --- | --- |
| **Ticker** | **Weight (Low Return Target)** | **Weight (High Return Target)** |
| TSLA | 0.02 | 0.25 |
| F | 0.10 | 0.05 |
| GM | 0.12 | 0.08 |
| AAPL | 0.15 | 0.07 |
| QCOM | 0.10 | 0.05 |
| SWKS | 0.10 | 0.06 |
| MSFT | 0.18 | 0.11 |
| GOOGL | 0.12 | 0.09 |
| NVDA | 0.05 | 0.20 |
| AMD | 0.06 | 0.04 |

From the analysis, we can observe that **portfolios targeting higher expected returns allocate more weight to high-beta, high-risk stocks** such as **Tesla (TSLA)** and **Nvidia (NVDA).** These stocks are more sensitive to market movements and have the potential to deliver higher returns, albeit with increased volatility.

Conversely, in the lower return target portfolio, the allocation shifts more towards stable, lower-beta stocks like **Microsoft (MSFT)** and **Apple (AAPL)**, reflecting a more conservative risk profile. This shift illustrates the natural trade-off between risk and return as investors seek greater returns, they must take on proportionally more market risk.

**10. Discussion and Interpretation**

The regression analysis shows that stock returns are strongly influenced by movements in the overall market, supporting the assumptions of the CAPM model.

The small alpha values suggest that most of the returns can be explained by market exposure, with little evidence of excess returns unrelated to market risk.

Stocks with higher beta values tend to offer higher expected returns, but they also bring more volatility to the portfolio, increasing overall risk.

The efficient frontier curves upward and outward, demonstrating the fundamental trade-off in Modern Portfolio Theory: achieving higher returns requires accepting greater risk.

By using quadratic programming, we can effectively allocate portfolio weights to minimise risk while still meeting specific return objectives, all within the given investment constraints.

**11. Conclusion**

This project provided a hands-on application of the Capital Asset Pricing Model (CAPM) and quadratic programming to tackle the challenge of portfolio optimisation. The workflow began with gathering and processing financial market data, followed by conducting CAPM regressions to estimate each asset’s expected return and sensitivity to market movements (beta). Using these results, a covariance matrix was built to reflect both market-driven (systematic) and asset-specific (idiosyncratic) risks.

Next, a constrained quadratic optimisation model was solved to determine the optimal mix of asset weights under return, investment, and no short-selling constraints. Finally, the efficient frontier was plotted to visually represent the trade-off between portfolio risk and expected return.

This approach offers a solid and systematic framework for constructing efficient portfolios. It can also be adapted to incorporate more complex financial models, additional constraints, or evolving market conditions.

**References**

* Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. The Journal of Finance, 19(3), 425–442.
* Markowitz, H. (1952). Portfolio Selection. The Journal of Finance, 7(1), 77–91.
* Yahoo Finance, yfinance Python library.